Estimation of wave run-up on smooth, impermeable slopes using the wave momentum flux parameter

Steven A. Hughes[*]

Research Hydraulic Engineer, US Army Engineer Research and Development Center, Coastal and Hydraulics Laboratory, 3909 Halls Ferry Road, Vicksburg, MI 39180-6199, USA

Abstract

This paper re-examines existing wave run-up data for regular, irregular and solitary waves on smooth, impermeable plane slopes. A simple physical argument is used to derive a new wave run-up equation in terms of a dimensionless wave parameter representing the maximum, depth-integrated momentum flux in a wave as it reaches the toe of the structure slope. This parameter is a physically relevant descriptor of wave forcing having units of force. The goal of the study was to provide an estimation technique that was as good as existing formulas for breaking wave run-up and better at estimating nonbreaking wave run-up. For irregular waves breaking on the slope, a single formula for the 2% run-up elevation proved sufficient for all slopes in the range $2/3 \leq \tan\alpha \leq 1/30$. A slightly different formula is given for nonbreaking wave run-up. In addition, two new equations for breaking and nonbreaking solitary maximum wave run-up on smooth, impermeable plane slopes are presented in terms of the wave momentum flux parameter for solitary waves. This illustrates the utility of the wave momentum flux parameter for nonperiodic waves.
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1. Introduction

Maximum wave run-up is an important design criterion for several types of coastal structures such as revetments, breakwaters and dikes. Also, beach processes such as beach/dune erosion and storm

flooding are related, in part, to wave run-up. Being able to estimate maximum wave run-up accurately can lead to more economical design. For example, the upper limit of expected wave run-up determines the crest elevation of a coastal structure designed to prevent wave overtopping. Overestimating maximum wave run-up could add significant cost to a rubble-mound breakwater. Below is a sampling of published papers related to wave run-up on coastal structures to give an idea of how predictive capability has

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[*] Tel.: +1 601 634 2026; fax: +1 601 634 3433.
E-mail address: Steven.A.Hughes@erdc.usace.army.mil.

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progressed since 1950. This is by no means a comprehensive overview of the run-up literature, and papers describing the more recent advances aimed at modeling wave run-up numerically have been excluded.

1.1. Regular wave run-up

Among the earlier investigations of wave run-up are the contributions of Granthem (1953), Saville (1955, 1956, 1958) and Savage (1958). These researchers measured wave run-up caused by regular wave trains impinging on various types of smooth and rough sloping structures, composite slope structures and other variations (stepped, recurved, etc.). Run-up results were plotted as functions of various wave parameters and structure slope, but no design formulas were given. Wave run-up estimation guidance was given in the earliest version of the Corps of Engineers' Shore Protection Manual (Beach Erosion Board, 1961) as a series of design nomograms, and this technique for regular wave run-up was propagated for over 20 years in essentially the same form (Shore Protection Manual, 1984), although extensions were made based on a reanalysis of regular wave run-up by Stoa (1978).

Early practical formulas for regular wave run-up on smooth and rough plane slopes and composite slopes were presented by Hunt (1959). Curiously, Hunt was a Major in the Corps of Engineers stationed in Detroit, MI, but his run-up formulas were never included in any version of the Corps' Shore Protection Manual. Hunt recognized that different formulas would be needed to differentiate run-up caused by nonbreaking waves that surge up steeper slopes from run-up caused by waves that break on milder slopes as plunging or spilling breakers.

For surging waves on plane, impermeable slopes, Hunt (1959) recommended simply

$$\frac{R}{H} \approx 3 \tag{1}$$

where R is the maximum vertical run-up from SWL and H is wave height (assumed to be the deepwater wave height, i.e., $H \approx H_o$). Hunt's analysis for the case where waves break on the slope resulted in a dimensionally nonhomogeneous equation for maximum run-up R given as

$$\frac{R}{H} = 2.3 \frac{\tan\alpha}{\sqrt{\dfrac{H}{T^2}}} \tag{2}$$

where T is wave period and α is structure slope angle. Recognizing that the coefficient 2.3 has units of $\text{ft}^{1/2}/\text{s}$, Eq. (2) can be expressed as a dimensionally homogeneous equation with the introduction of the gravity constant in imperial units, i.e.,

$$\frac{R}{H} = 1.0 \frac{\tan\alpha}{\sqrt{H/L_o}} \quad \text{or} \quad \frac{R}{H} = 1.0 \xi_o \tag{3}$$

where the deepwater wavelength is given by $L_o = (g/2\pi)T^2$, and

$$\xi_o = \frac{\tan\alpha}{\sqrt{H/L_o}} \tag{4}$$

is defined as the deepwater Iribarren number (Iribarren and Nogales, 1949), also known as the "surf similarity parameter" (Battjes, 1974a). Often the parameter ξ_o is calculated using a finite-depth local wave height in the vicinity of the slope toe rather than a true deepwater H_o. For example, in laboratory experiments, it is common to specify H as the wave height measured over the flat-bottom portion of the wave facility before significant wave transformation occurs due to shoaling. In some cases, $H \approx H_o$, but this is not always assured. For the discussion in this paper, we will assume that ξ_o is based on the local wave height at or near the toe of the slope rather than H_o.

1.2. Irregular wave run-up

The capability to predict maximum wave run-up on a variety of structure slopes and surface types advanced structure design, but the above regular wave methods were not altogether realistic given the irregular character of natural sea states. The importance of irregular wave run-up on structures was acknowledged in the 1977 and 1984 editions of the SPM (Shore Protection Manual, 1977, 1984). Based on earlier publications suggesting irregular wave run-

up for wind generated waves is Rayleigh distributed, the SPM proposed a Rayleigh cumulative distribution for estimating run-up given by

$$\frac{R_P}{R_s} = \left(-\frac{\ln P}{2} \right)^{1/2} \tag{5}$$

where P is the exceedence probability, R_P is the run-up elevation associated with P and R_s is the significant wave run-up. In other words, the run-up level exceeded by 2% of the run-ups would be estimated with $P=0.02$ and denoted as $R_{0.02}$. The SPM recommended R_s be estimated as the regular wave run-up value determined from the existing nomogram procedures.

For many years, the Netherlands used a simple formula for estimating irregular wave run-up given by Wassing (1957) as

$$R_{u2\%} = 8H_{1/3} \tan \alpha \tag{6}$$

where $R_{u2\%}$ is the vertical elevation from SWL exceeded by 2% of the run-ups and $H_{1/3}$ is the significant wave height (average of the highest 1/3 waves) at the toe of the structure slope. This formula was valid for milder slopes with $\tan\alpha \leq 1/3$.

Battjes (1974b) demonstrated the applicability of the Hunt formula (Eq. (3)) for irregular waves breaking as plungers on the slope for the 2% run-up level with the formulation

$$\frac{R_{u2\%}}{H_{1/3}} = C_m \xi_{om} \quad \text{where} \quad \xi_{om} = \frac{\tan\alpha}{\sqrt{H_{1/3}/L_{om}}} \tag{7}$$

and L_{om} is the deepwater wavelength associated with the mean irregular wave period, T_m. Battjes reported the coefficient C_m varied from 1.49 for fully developed seas to 1.87 for seas in the initial stages of development. Prototype measurements by Grüne (1982) expanded the range of C_m to between 1.33 and 2.86. Van der Meer and Stam (1992) converted Eq. (7) to a slightly different form

$$\frac{R_{u2\%}}{H_{1/3}} = C_p \xi'_{op} \quad \text{where} \quad \xi'_{op} = \frac{\tan\alpha}{\sqrt{H_{1/3}/L_{op}}} \tag{8}$$

and L_{op} is the deepwater wavelength associated with the peak spectral wave period, T_p. The range given by Battjes for the coefficient C_m was converted to $1.3 \leq C_p \leq 1.7$ by assuming the ratio T_p/T_m is approximately 1.1–1.2. Van der Meer and Stam also noted

the formula should be restricted to values of $\xi'_{op} < 2$, and they demonstrated good correspondence to Eq. (8) for large-scale laboratory run-up experiments with plunging breakers on slopes between $\tan\alpha = 1/3 - 1/8$. Battjes (1974b) also included reduction factors to account for various rough slopes (rock, concrete armor, etc.) and his table was included in the SPM.

Ahrens (1981) conducted laboratory measurements of irregular wave run-up on smooth, impermeable plane slopes with slope angles ranging between $\tan\alpha = 1/1$ and $\tan\alpha = 1/4$. For the mildest 1-on-4 slope where most of the waves broke on the slope as plunging breakers, Ahrens proposed the run-up elevation exceeded by 2% of the run-ups be estimated using the Hunt formula (Eq. (3)), i.e.,

$$\frac{R_{u2\%}}{H_{mo}} = 1.6 \xi_{op} \tag{9}$$

where

$$\xi_{op} = \frac{\tan\alpha}{\sqrt{H_{mo}/L_{op}}} \tag{10}$$

and H_{mo} is the energy-based zeroth-moment wave height. The coefficient 1.6 is near the high limit given by Battjes (1974b). Similar formulas were given for significant run-up and mean run-up. For steeper slopes, Ahrens gave an expanded equation in the form

$$\frac{R_X}{H_{mo}} = C_1 + C_2 \left(\frac{H_{mo}}{gT_p^2} \right) + C_3 \left(\frac{H_{mo}}{gT_p^2} \right)^2 \tag{11}$$

where R_X is a place-holder for run-up exceedence levels (2%, significant and mean), and regression coefficients C_1, C_2 and C_3 are tabulated for different slopes and exceedence levels.

In Ahrens (1981), significant wave height was denoted as H_s, but this value was calculated from the measured wave spectra according to the definition for H_{mo} (Ahrens, personal communication). This raises an interesting point with respect to design formula that use the notation H_s to represent irregular waves. Unless explicitly stated, the user cannot be certain whether H_s means the time-series wave parameter $H_{1/3}$ or the frequency-domain parameter H_{mo}. For narrow-band spectra where the wave heights can be assumed Rayleigh distributed, $H_{1/3} \approx H_{mo}$, and it matters not which parameter is used for H_s in the design formula. However, as waves approach incipient breaking, $H_{1/3}$

becomes larger than H_{mo} and the design formulas will give different answers. Thus, it is important to determine which parameter is meant by the notation H_s.

Ahrens et al. (1993) examined a large set of run-up data for smooth, impermeable slopes ranging between $\tan\alpha=1/1$–$1/4$, and they proposed design formulas for significant and 2% run-up elevations. These formulas were then compared to other published data sets, and modifications to the formulas were made where necessary. The final recommended equation for breaking waves (plunging or spilling on the slope) was given as

$$\frac{R_{u2\%}}{H_{mo}} = \frac{2.26\xi_{op}}{\left(1 + 0.324\xi_{op}\right)} \quad \text{for} \quad \left(\xi_{op}\leq2.5\right) \qquad (12)$$

with ξ_{op} based on H_{mo} and T_p. The appearance of ξ_{op} in both the numerator and denominator has no physical meaning, it is simply an empirical fit to the data. For nonbreaking waves (surging/collapsing), Ahrens et al. recommended

$$\frac{R_{u2\%}}{R_s} = 1.6\pm0.24 \quad \text{for} \quad \left(\xi_{op}\geq4.0\right) \qquad (13)$$

with the significant run-up estimated by

$$\frac{R_s}{H_{mo}} = \exp\left[2.48X_p + 0.446(\cos\alpha)^{3.5} + 0.19\Pi\right]$$
$$\text{for} \quad \left(\xi_{op}\geq4.0\right) \qquad (14)$$

where

$$X_p = \left(\frac{h\cot\alpha}{L_p}\right) - \left(\frac{h\cot\alpha}{L_p}\right)^2 \text{ and } \Pi = \frac{H_{mo}/L_p}{\left[\tanh\left(\frac{2\pi h}{L_p}\right)\right]^3} \qquad (15)$$

and h is the water depth at the toe of the structure, and L_p is the local wavelength associated with the spectral peak period T_p. In the transitional range $2.5<\xi_{op}<4.0$, a weighted average technique was proposed. A key point made by Ahrens et al. (1993) was the lack of wave run-up data representing severe wave conditions ($0.33\leq H_{mo}/h\leq0.60$) relative to water depth at the structure toe which may be the design condition for

shallow water structures. Thus, there could be uncertainty regarding wave run-up formula accuracy under these conditions.

In the recently available Coastal Engineering Manual (CEM) (Burcharth and Hughes, 2002), two sets of design guidance are presented for irregular wave run-up on smooth, impermeable slopes. For steeper slopes in the range $\tan\alpha=1/1$–$1/4$, the data of Ahrens (1981) was represented by

$$\frac{R_{u2\%}}{H_{mo}} = 1.6\xi_{op} \quad \text{for} \quad \left(\xi_{op}\leq2.5\right) \qquad (16)$$

and

$$\frac{R_{u2\%}}{H_{mo}} = 4.5 - 0.2\xi_{op} \quad \text{for} \quad \left(2.5<\xi_{op}<9\right) \qquad (17)$$

Once again, the irregular wave deepwater Iribarren number is based on T_p and H_{mo} at, or near, the toe of the slope. For milder structure slopes in the range $\tan\alpha=1/3$–$1/8$, the CEM recommends the guidance of De Waal and Van der Meer (1992), given by

$$\frac{R_{u2\%}}{H_{1/3}} = 1.5\xi'_{op} \quad \text{for} \quad \left(0.5<\xi'_{op}\leq2.0\right) \qquad (18)$$

and

$$\frac{R_{u2\%}}{H_{1/3}} = 3.0\xi'_{op} \quad \text{for} \quad \left(2.0<\xi'_{op}<4\right) \qquad (19)$$

with ξ'_{op} calculated using T_p and $H_{1/3}$. De Waal and Van der Meer stated water depth at the toe of the structure was at least three times $H_{1/3}$ for all data used to establish Eqs. (18) and (19) so they assumed waves were Rayleigh distributed. Therefore, it should be reasonable to apply Eqs. (18) and (19) using H_{mo} instead of $H_{1/3}$. De Waal and Van der Meer (1992) also developed guidance for run-up on composite slopes (berm), and they gave reduction factors for slope roughness, shallow water and incident wave angle.

All of the wave run-up studies discussed above pertain to run-up on coastal structures with slopes as mild as $\tan\alpha=1/8$. Not as many laboratory experiments have been conducted for more gentle slopes similar to those found on natural beaches.

Mase (1989) presented results from irregular wave run-up experiments on mild impermeable plane slopes of $\tan\alpha=1/5$, $1/10$, $1/20$ and $1/30$. He

expressed the 2%-run-up elevation as the following function of ξ_{op}

$$\frac{R_{u2\%}}{H_{mo}} = 1.86 (\xi_{op})^{0.71} \quad \text{for} \quad (\xi_{op} \leq 3.0) \tag{20}$$

Mase also presented empirical estimation formulas of the same form for R_{max}, $R_{1/10}$, $R_{1/3}$ and $\bar{R}$ with different coefficients and slightly different exponents. It was noted in the Coastal Engineering Manual (Smith, 2002) that Eq. (20) overpredicts the best-fit line to Holman's (1986) field measurements of beach run-up by a factor of two, but the equation did provide an approximate upper envelope to the measurements. Effects of beach permeability, nonuniform beach slope, wave breaking over sand bars and other factors related to Holman's data were not quantified, and these could explain the differences between laboratory and field data.

Douglass (1992) analyzed Holman's (1986) field measurements of beach run-up and argued that beach slope was not an important parameter for predicting wave run-up on natural beaches. Plots of relative maximum run-up versus Iribarren number showed no better correlation than plots of maximum relative run-up versus wave steepness. Furthermore, maximum relative run-up plotted as a function of beach slope exhibited little correlation. Given the problem of defining beach slope and the slope variability, Douglass suggested that beach slope be eliminated from the run-up equation when applied to beaches, and he proposed the following equation based on Holman's data

$$\frac{R_{max}}{H_{mo}} = \frac{0.12}{\sqrt{\dfrac{H_{mo}}{L_{op}}}} \tag{21}$$

1.3. Importance of Iribarren number

Practically, all present day wave run-up guidance is given in terms of the deepwater Iribarren number using local wave height, and there can be no question about its significance when waves break as plunging or spilling waves on the slope. However, run-up data for nonbreaking breaking waves that surge up steeper slopes does not correlate as well to the Iribarren number, and instead run-up appears in this case to be directly related to wave height. Rearranging Hunt's

equation for breaking wave run-up, we see run-up is directly proportional to wave period, slope and the square root of wave height, i.e.,

$$R \propto H^{1/2} T \tan \alpha \tag{22}$$

Thus, variation in incident wave height is less important and water depth at the toe of the structure slope is not included. A possible explanation for the success of the Hunt formulation for breaking waves may lie in the assumption that broken waves become selfsimilar during shoaling. Consider two waves having significantly different wave heights but the same value of wave steepness, H/L_o. Depth-limited breaking will occur at different water depths on the slope, and the magnitude of the dimensional flow kinematic parameters at breaking will be different between the two waves. However, the good correlation between run-up and deepwater Iribarren number suggests that depth of initial wave breaking and breaking wave kinematics are not critical for breaking wave run-up because ultimately the two different waves having the same value of H/L_o become similar in the surf zone as observed by Battjes (1974a).

For nonbreaking wave run-up, we should expect wave kinematics to be more important, particularly for shallow water nonlinear waves approaching limiting steepness. Wave steepness contained in the deepwater Iribarren number (H/L_o) does not adequately characterize wave nonlinearity in shallow water, so we might expect poorer results when using ξ_o to estimate nonbreaking wave run-up. It is anticipated that water depth at the structure toe will become an important parameter for nonbreaking wave run-up such as shown by Ahrens et al. (1993) in Eqs. (14) and (15).

1.4. Present study

This paper re-examines existing wave run-up data for regular, irregular and solitary waves on smooth, impermeable plane slopes. A crude model is used to derive a new wave run-up equation in terms of a dimensionless wave parameter (Hughes, 2004) representing the maximum, depth-integrated momentum flux in a wave as it reaches the toe of the structure slope. The goal of the study was to provide an estimation technique that was as good as existing formulas for breaking wave run-up and better at

estimating nonbreaking wave run-up. For irregular waves breaking on the slope, a single formula proved sufficient for all slopes in the range $2/3 \leq \tan\alpha \leq 1/30$. A slightly different formula is given for nonbreaking wave run-up. In addition, two new equations for breaking and nonbreaking solitary wave run-up are presented.

2. Wave momentum flux parameter

Hughes (2004) introduced a wave parameter representing the maximum depth-integrated wave momentum flux occurring in a wave of permanent form, i.e., the maximum over the wave of the integral

$$M_{\mathrm{F}}(x,t) = \int_{-h}^{\eta(x)} \left(p_{\mathrm{d}} + pu^2 \right) \mathrm{d}z \qquad (23)$$

where $M_{\mathrm{F}}(x,t)$—depth-integrated wave momentum flux at x and t; p_{d}—instantaneous wave dynamic pressure at a specified position; u—instantaneous horizontal water velocity at the same specified position; ρ—water density; h—water depth; x—horizontal direction perpendicular to wave crests; z—vertical direction, positive upward with $z=0$ at still water level; $\eta(x)$—sea surface elevation at location x; t—time.

Maximum depth-integrated wave momentum flux has units of force per unit wave crest, and Hughes speculated this wave parameter may prove useful in empirical correlations relating waves to nearshore coastal processes occurring on beaches and coastal structures. He also noted that integration of Eq. (23) over a uniform periodic wave results in radiation stress, S_{xx}, as introduced by Longuet-Higgins and Stewart (1964). However, values of M_{F} vary over a wave from large positive values at the wave crest to large negative values in the trough, whereas the value of S_{xx} is relatively small in comparison to the maximum. When considering force loading on coastal structures, perhaps better correlations can be made using a parameter representative of the maximum force in the wave instead of one corresponding to the integration over the entire wavelength.

Estimates of $(M_{\mathrm{F}})_{\max}$ can be made for any wave for which sea surface elevation and wave kinematics are known either through theory or measurement. Hughes

(2004) derived formulas for estimating the maximum depth-integrated wave momentum flux for periodic (regular) waves and solitary waves.

2.1. Estimates for periodic waves

An estimate of wave momentum flux in periodic waves was given by Hughes for first-order wave theory in nondimensional form as

$$\left(\frac{M_{\mathrm{F}}}{\rho g h^2} \right)_{\max} = \frac{1}{2} \left(\frac{H}{h} \right) \frac{\tanh kh}{kh}$$
$$+ \frac{1}{8} \left(\frac{H}{h} \right)^2 \left[1 + \frac{2kh}{\sinh 2kh} \right] \qquad (24)$$

The dimensionless parameter to the left of the equal sign represents the nondimensional maximum depth-integrated wave momentum flux, and it is referred to as the "wave momentum flux parameter".

Eq. (24) expresses nondimensional maximum wave momentum flux as a function of relative wave height (H/h) and relative depth (kh). However, integration over the water depth stopped at the still water level, and Eq. (24) does not include that part of the wave above the still water level where a significant portion of the wave momentum flux is found.

An improved estimate of $(M_{\mathrm{F}})_{\max}$ was obtained using extended-linear theory in which expressions for linear wave kinematics are assumed to be valid in the crest region so the integration could be continued up to the free surface at the crest. This resulted in a slightly different expression given by

$$\left(\frac{M_{\mathrm{F}}}{\rho g h^2} \right)_{\max}$$
$$= \frac{1}{2} \left(\frac{H}{h} \right) \frac{\sinh[k(h + H/2)]}{kh \cosh(kh)}$$
$$+ \frac{1}{8} \left(\frac{H}{h} \right)^2 \left[\frac{\sinh[2k(h + H/2) + 2k(h + H/2)]}{\sinh 2kh} \right] \qquad (25)$$

However, the wave form is still sinusoidal rather than having peaked crests and shallow troughs typical of nonlinear shoaled waves and, consequently, the extended-linear theory under-predicts momentum flux
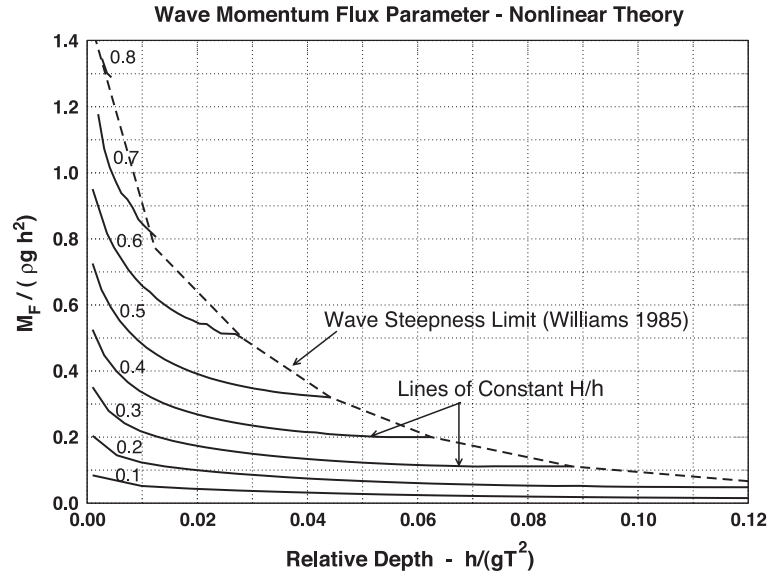
Wave Momentum Flux Parameter - Nonlinear Theory

Fig. 1. Wave momentum flux parameter versus h/gT^2 (Fourier wave theory).

under the crest when waves become nonlinear. For both linear and extended-linear theory, the relative contribution of the horizontal velocity term (ρu^2) to the total wave momentum flux varies between about 5% for low amplitude, long period waves to nearly 30% for waves approaching limiting steepness.

To better represent the maximum depth-integrated wave momentum flux in nonlinear waves, Hughes (2004) used Fourier approximation wave theory for regular steady waves over a horizontal bottom. He determined wave kinematics and calculated values of the dimensionless wave momentum flux parameter for selected combinations of relative wave height (H/h) and relative water depth (h/gT^2). Results were plotted as a family of curves representing constant values of H/h as shown on Fig. 1.

The dashed line on Fig. 1 gives the steepness-limited wave breaking criterion tabulated by Williams (1985) and expressed by Sobey (1998) as the rational approximation

$$\frac{\omega^2 H_{\text{limit}}}{g} = c_o \tanh\left(\frac{a_1 r + a_2 r^2 + a_3 r^3}{1 + b_1 r + b_2 r^2}\right) \qquad (26)$$

where $r = \omega^2 h/g$, $a_1 = 0.7879$, $a_2 = 2.0064$, $a_3 = -0.0962$, $b_1 = 3.2924$, $b_2 = -0.2645$ and $c_o = 1.0575$. Sobey noted

the above expression has a maximum error of 0.0014 over the range of Williams' table. Williams' (1985) tabulation of limit waves is more accurate than the traditional limit wave steepness given by

$$\frac{H_{\text{limit}}}{L} = 0.412 \tanh(kh) \qquad (27)$$

which overestimates limiting steepness for long waves and underestimates limiting steepness for short waves.

An empirical equation for estimating the wave momentum flux parameter for finite amplitude steady waves was established using the calculated curves of constant H/h shown in Fig. 1. A nonlinear best-fit of a two parameter power curve was performed for each calculated H/h curve, and the resulting coefficients and exponents for each fitted power curve were also approximated as power curves. The resulting, purely empirical, equation representing the curves of constant H/h shown on Fig. 1 is given as

$$\left(\frac{M_F}{\rho g h^2}\right)_{\text{max}} = A_0 \left(\frac{h}{gT^2}\right)^{-A_1} \qquad (28)$$

where

$$A_0 = 0.6392 \left(\frac{H}{h} \right)^{2.0256} \tag{29}$$

$$A_1 = 0.1804 \left(\frac{H}{h} \right)^{-0.391} \tag{30}$$

Even though the empirical coefficients and exponents in Eqs. (29) and (30) are expressed to four decimal places, corresponding accuracy is not implied. Rounding to two decimal places should be reasonably adequate for practical application of these empirical equations.

The empirical equation represented by Eq. (28), along with Eqs. (29) and (30), provides an easy method for estimating maximum wave momentum flux for finite amplitude, steady, regular waves. This formulation gives more accurate estimates of the true maximum depth-integrated wave momentum flux than linear and extended linear theory because it better represents the momentum flux in the wave crest, which is expected to be critical for most applications to coastal structures.

For irregular wave trains, Hughes recommended that the wave momentum flux parameter be represented by substituting frequency-domain irregular wave parameters H_{mo} (zeroth-moment wave height) and T_p (peak spectral period) directly into the empirical Eqs. (28)–(30). While this might not be the best set of irregular wave parameters to use, these frequency-domain parameters are commonly reported for laboratory and field measurements, and numerical irregular wave hindcast and forecast models output frequency-domain parameters.

2.2. Estimates for solitary waves

Hughes (2004) also derived an expression for the nondimensional wave momentum flux parameter using first-order solitary wave theory given as

$$\left(\frac{M_F}{\rho g h^2} \right)_{max} = \frac{1}{2} \left[\left(\frac{H}{h} \right)^2 + 2 \left(\frac{H}{h} \right) \right]$$

$$+ \frac{N^2}{2M} \left(\frac{H}{h} + 1 \right) \left\{ \tan \left[\frac{M}{2} \left(\frac{H}{h} + 1 \right) \right] \right.$$

$$+ \frac{1}{3} \tan^3 \left[\frac{M}{2} \left(\frac{H}{h} + 1 \right) \right] \right\} \tag{31}$$

with the coefficients M and N approximated by the empirically determined functions

$$M = 0.98 \left\{ \tanh \left[2.24 \left(\frac{H}{h} \right) \right] \right\}^{0.44} \tag{32}$$

$$N = 0.69 \tanh \left[2.38 \left(\frac{H}{h} \right) \right] \tag{33}$$

Note that maximum depth-integrated wave momentum flux for solitary waves is a function of only relative wave height, H/h. The first bracketed term in Eq. (31) arises from the dynamic pressure, and the second term represents the contribution of horizontal velocity to the maximum wave momentum flux.

The solitary wave estimates of the wave momentum flux parameter represent the upper limit of the nonlinear (Fourier) wave case when $h/(gT^2)$ approaches zero (see Fig. 1). At a value of $H/h=0.1$, the velocity term contributes only about 7% of the calculated momentum flux, whereas as at $H/h=0.8$ the percentage increases to around 38% of the total.

3. Wave run-up as a function of wave momentum flux parameter

In the following sections the maximum, depth-integrated wave momentum flux parameter is correlated to existing available data of normally incident, breaking and nonbreaking wave run-up on smooth, impermeable plane slopes. Included are data for regular waves, irregular waves and solitary waves.

3.1. Wave run-up deviation

Archetti and Brocchini (2002) showed a strong correlation between the time series of wave run-up on a beach and the time series of depth-integrated mass flux within the swash zone. They also noted that the local depth-integrated momentum flux was balanced mainly by the weight of water in the swash zone, which was approximated as a triangular wedge. Their observation suggests that maximum wave run-up on an impermeable slope might be directly proportional to the maximum depth-integrated wave momentum flux contained in the wave before it reaches the toe of

the slope. In this section, a simple derivation is performed based on this assumption.

Fig. 2 depicts a simplification of wave run-up geometry at the point of maximum wave run-up. The force of the wave has "pushed" the water up the impermeable slope. At the instant of maximum run-up, the fluid within the hatched area on Fig. 2 has almost no motion. (Li and Raichlen, 2002 noted this was nearly the case for solitary wave run-up.) Following the lead of Archetti and Brocchini (2002), a simple physical argument is that the weight of the fluid contained in the hatched wedge area ABC ($W_{(ABC)}$) is proportional to the maximum depth-integrated wave momentum flux of the wave before it reached the toe of the structure slope, i.e.,

$$K_P (M_F)_{max} = K_M W_{(ABC)} \qquad (34)$$

where K_M is an unknown constant of proportionality and K_P is a reduction factor to account for slope porosity ($K_P=1$ for impermeable slopes).

The weight of water per unit width contained in triangle ABC shown on Fig. 2 is given by

$$W_{(ABC)} = \frac{\rho g}{2} \frac{R^2}{\tan\alpha} \left[\frac{\tan\alpha}{\tan\theta} - 1 \right] \qquad (35)$$

where R is maximum vertical run-up elevation from SWL, α is structure slope angle, and θ is an unknown angle between still water level and run-up water surface (which is assumed to be a straight line). Substituting Eq. (35) into Eq. (34), rearranging and dividing both sides by h^2 yields a new run-up

equation based on the dimensionless maximum wave momentum flux parameter, i.e.,

$$\frac{R}{h} = \left(\frac{2 K_P \tan\alpha}{K_M \left[\frac{\tan\alpha}{\tan\theta} - 1 \right]} \right)^{1/2} \left[\frac{M_F}{\rho g h^2} \right]^{1/2} \qquad (36)$$

or more simply

$$\frac{R}{h} = C F(\alpha) \left[\frac{M_F}{\rho g h^2} \right]^{1/2} \qquad (37)$$

where C is an unknown constant and $F(\alpha)$ is a function of slope angle to be determined empirically. For convenience, the "max" subscript has been dropped from the wave momentum flux parameter.

In the new run-up equation relative run-up (R/h) is directly proportional to the square root of the wave momentum flux parameter. Representing the run-up sea surface slope as a straight line is an approximation, but, for waves on gentle slopes were wave breaking has occurred, this might be a reasonable assumption as shown by Li (2000). On steeper slopes where waves behave more like surging breakers, the sea surface elevation will have more of a concave shape, also illustrated by Li. Another simplification in this derivation is the absence of slope friction, and this was shown by Archetti and Brocchini (2002) to be important for swash zone run-up processes on mild slopes where the wave travels over a much longer distance.

3.2. Regular wave run-up

The proposed run-up relationship given by Eq. (37) was empirically fit to existing regular wave run-up laboratory test results published many years ago by Granthem (1953) and Saville (1955). In Granthem's impermeable-slope tests, waves propagated over a flat bottom before reaching a linear slope that was varied between 15° and vertical. Wave heights were somewhat mild with maximum relative wave height of $H/h \approx 0.35$. A total of 52 run-up values for slopes ranging over $\cot\alpha$=1.00, 1.43, 1.73, 2.14, 2.75 and 3.73 are used in this reanalysis.

Most of the waves in the experiments reported by Saville (1955) propagated over a mild sloping bottom before reaching the impermeable structure slope. Run-up values were reported for slopes with $\cot\alpha$=2.0, 3.0,
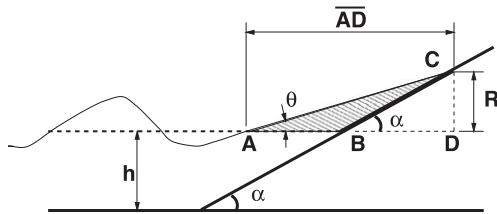
Fig. 2. Maximum wave run-up on a smooth impermeable plane slope.

4.0, 6.0 and 10.0 for a total of 100 observations. For purposes of this paper, wave height at the structure toe was estimated by linear shoaling of the measured wave height recorded at the offshore measurement location. Generally, waves were significantly larger than in Granthem's tests and only those waves with $H/h < 1.0$ at the toe after shoaling were used for the present analysis.

The wave momentum flux parameter was estimated for all 152 observations using the formula given by Eq. (28), even though application to waves with $H/h \geq 0.8$ is not strictly appropriate. A best-fit of Eq. (37) to the run-up data set yielded a reasonably simple equation,

$$\frac{R}{h} = 3.84 \tan\alpha \left(\frac{M_F}{\rho g h^2} \right)^{1/2} \quad (38)$$

where ($3.84 \tan\alpha$) represents the empirical function $CF(\alpha)$ in Eq. (37). The results are shown in Fig. 3 with the straight line representing Eq. (38). Granthem's data are solid markers and Saville's data are hollow markers. Most of the data follow the trend given by the straight line with the exception of Granthem's results for a 1:1 slope, which have much lower run-up values than estimated. Waves on this steep slope were probably surging breakers or nonbreaking waves, which do not conform to the straight-line sea surface

assumed in the derivation of Eq. (37). The root-mean-squared error between measured and predicted R/h was 0.19, excluding Granthem's 1:1 slope data.

For comparison, the same data have been plotted on Fig. 4 using Hunt's Eqs. (1) and (3). (Note that the ordinate axis is run-up nondimensionalized by wave height rather than water depth.) The Iribarren number characterizes run-up very well for mild slopes that produce low ξ_o values implying plunging or spilling wave breaking on the slope. As slopes get steeper and ξ_o exceeds 2.0, scatter becomes greater. Granthem's 1:1 slope data are problematic in this plot as well.

3.3. Irregular wave run-up

Two published irregular wave laboratory data sets were used to test the simple run-up relationship given by Eq. (37) for irregular wave run-up. Both data sets represent normally incident wave run-up on smooth, impermeable plane slopes. Fig. 5 shows irregular breaking and nonbreaking wave run-up data for 275 values of 2% run-up elevation measured by Ahrens (1981) versus Iribarren number for steeper slopes ranging between $1/4 \leq \tan\alpha \leq 1/1$. The solid lines on Fig. 5 are the recommended prediction equations (Eqs. (16) and (17)) given in the Coastal Engineering Manual. Data corresponding to milder slopes are
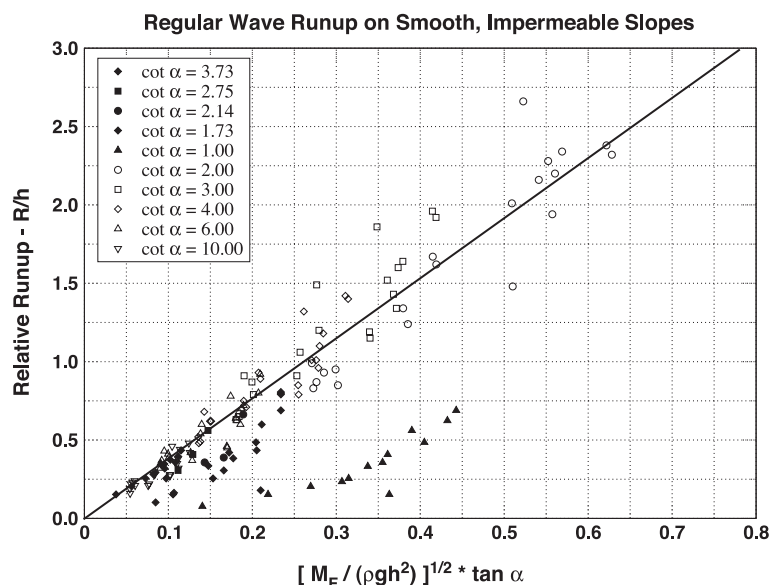

Fig. 3. Regular wave relative run-up versus Eq. (38).

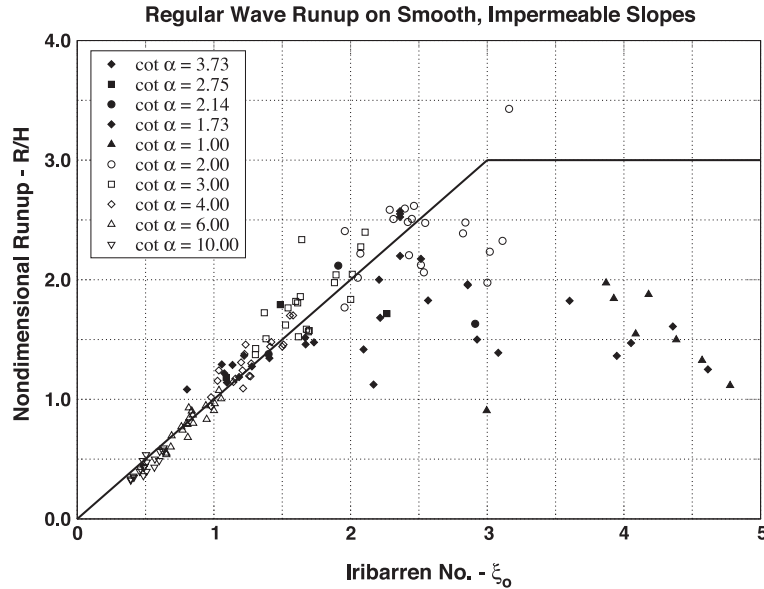Regular Wave Runup on Smooth, Impermeable Slopes

Fig. 4. Nondimensional run-up versus Iribarren number for regular waves.

clustered reasonably well for values of Iribarren number below about 3.0. At values of $\xi_{op} > 3.0$ (representing steeper slopes and/or longer nonbreaking waves) scatter increases significantly. Ahrens et al. (1993) discussed reasons for the scatter and proposed modified equations (Eqs. (12)–(15)) to reduce the scatter for nonbreaking wave conditions.

Mase (1989) listed 120 values of 2%-run-up elevation for milder slopes ranging between 1/30 $\leq \tan\alpha \leq 1/5$. Fig. 6 plots nondimensional 2% run-up $R_{u2\%}/H_{mo}$ versus Iribarren number ξ_{op}. The solid line is Mase's best fit to the data previously given by Eq. (20). A very good fit is seen for all but a few points for the steepest slope at $\xi_{op} > 2.5$.

Irregular Wave Runup on Smooth, Impermeable Slopes
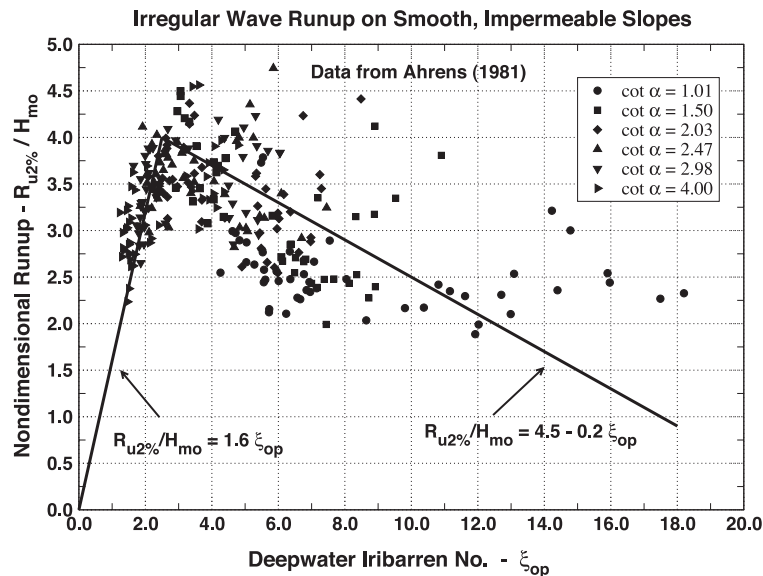
Fig. 5. Ahrens' (1981) original 2% run-up data plotted versus ξ_{op}.
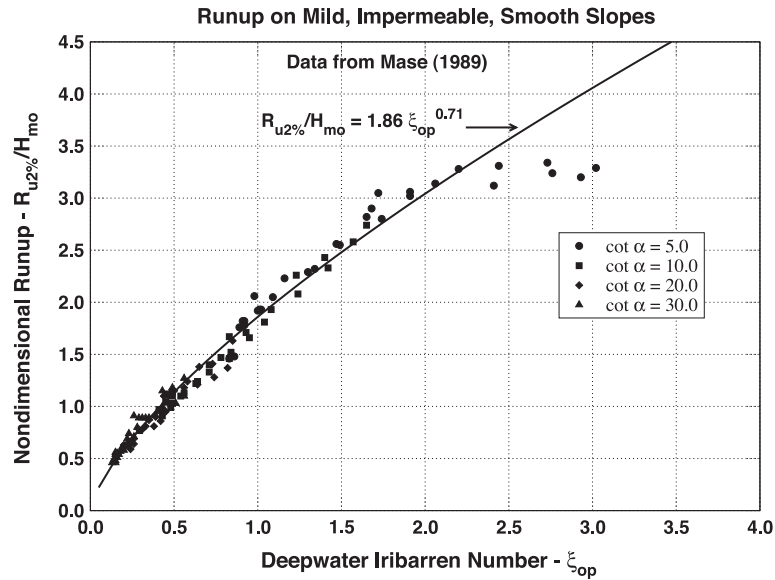
Fig. 6. Mase's (1989) data plotted versus Iribarren number.

Application of maximum depth-integrated wave momentum flux parameter to irregular waves requires substitution of representative irregular wave parameters H_{mo} and T_p for the regular wave height H and period T, respectively, in Eqs. (28)–(30). Original data provided by Ahrens included his measurements of H_{mo} and T_p. Mase (1989) did not directly tabulate T_p

so it was necessary to extract T_p from given values of ξ_{op} using H_{mo} and $\tan\alpha$. (Mase's listed values of H_{mo}/L_{op} appeared to have been rounded off and would have produced a less reliable estimate of T_p).

Ahrens' (1981) and Mase's (1989) measurements for $R_{u2\%}$ were normalized by water depth h at the toe of the plane slope and plotted versus the
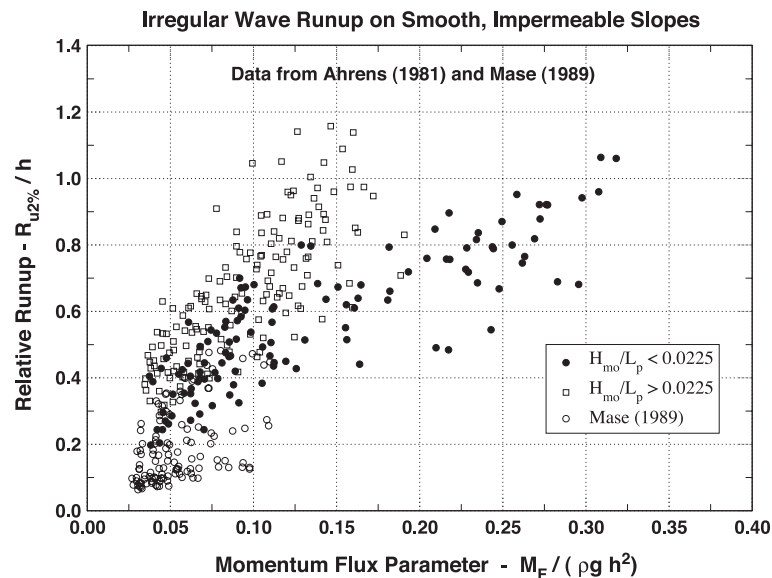
Fig. 7. Ahrens' (1981) and Mase's (1989) data plotted versus wave momentum flux parameter.

calculated wave momentum flux parameter as shown on Fig. 7. Ahrens' data exhibited two distinct trends that seemed to be delineated by a value of local spectral steepness corresponding to $H_{mo}/L_p=0.0225$ regardless of structure slope over the range of tested slopes. This steepness value appears to represent transition of breaker type from nonbreaking/surging/collapsing waves for $H_{mo}/L_p<0.0225$ to plunging/spilling waves when $H_{mo}/L_p>0.0225$. Physically, the data indicate that nonbreaking/surging/collapsing waves need more wave momentum flux than plunging/spilling waves to achieve the same 2% run-up level on the same slope and water depth. At slopes of $\tan\alpha=1/4$ and milder, which includes all of Mase's data, there was no differentiation based on wave steepness. This implies that most of the irregular waves were breaking as plunging or spilling waves on the milder slopes.

The irregular wave run-up data from Ahrens were separated into two groups according to the steepness criterion $H_{mo}/L_p=0.0225$. Mase's data were all assumed to represent breaking waves. The data were then further divided into groups according to slope so that a best-fit expression for the slope function $CF(\alpha)$ in Eq. (37) could be determined. The best-fit points for each slope, and the resulting slope functions

for all nonbreaking and breaking waves, are shown on Fig. 8. The corresponding empirical run-up equations are given as.

Nonbreaking/surging/collapsing waves ($H_{mo}/L_p<0.0225$):

$$\frac{R_{u2\%}}{h} = 1.75\left(1 - e^{-[1.3\cot\alpha]}\right)\left[\frac{M_F}{\rho gh^2}\right]^{1/2}$$

for $1/4\leq\tan\alpha\leq1/1$ (39)

Plunging/spilling waves ($H_{mo}/L_p>0.0225$):

$$\frac{R_{u2\%}}{h} = 4.4(\tan\alpha)^{0.7}\left[\frac{M_F}{\rho gh^2}\right]^{1/2}$$

for $1/5\leq\tan\alpha\leq2/3$ (40)

Plunging/spilling waves (any value of H_{mo}/L_p):

$$\frac{R_{u2\%}}{h} = 4.4(\tan\alpha)^{0.7}\left[\frac{M_F}{\rho gh^2}\right]^{1/2}$$

for $1/30\leq\tan\alpha\leq1/5$ (41)

Data for slope $\cot\alpha=1.01$ and $H_{mo}/L_p>0.0225$ did not follow the trend found for the other slopes,
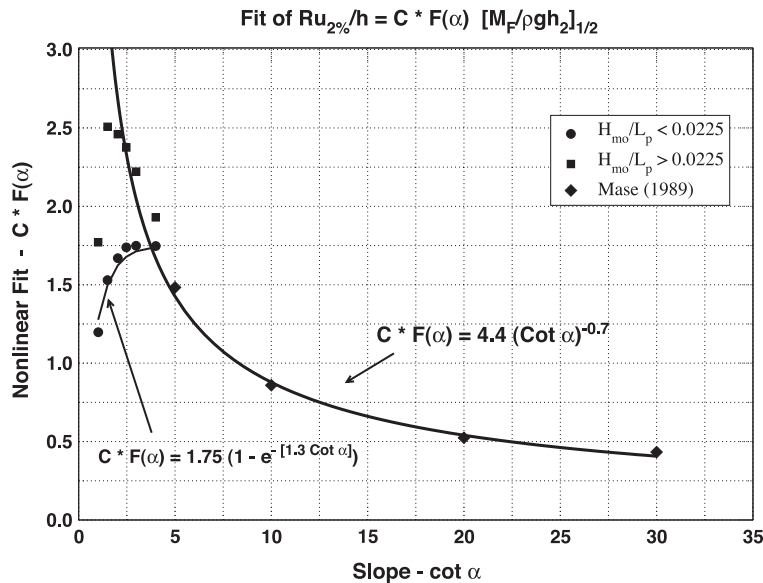

Fig. 8. Empirical slope functions for breaking and nonbreaking irregular wave run-up.

and thus, were excluded from the empirical formulation as indicated by the range of applicability on Eq. (40). This data set is evident on Fig. 8. One possible explanation is that these shorter waves on the steep 1:1 slope produced a run-up wedge with a concave sea surface profile that was not well approximated by the straight-line water surface hypothesized in Fig. 2. Thus, the derived run-up Eq. (37) is not appropriate.

Fig. 9 compares predictions based on Eqs. (39)–(41) to Ahrens' and Mase's observed 2% run-up values. Mase's data are the hollow circles clustered toward the lower left corner of the plot. With the exception of data for slope $\cot\alpha=1.01$ and $H_{mo}/L_p>0.0225$ (shown by the X-symbol), the prediction is reasonable. For comparison, Fig. 10 plots Ahrens' (1981) measurements of $R_{u2\%}/H_{mo}$ versus estimates using the prediction equations for steeper slopes given in the CEM (Eqs. (16) and (17)). The new irregular wave run-up equations exhibit less scatter for the Ahrens' data set then the prediction equations recommended in the CEM.

A single equation, given by either Eq. (40) or (41), was found to work reasonably well for breaking waves over the entire range of slopes from
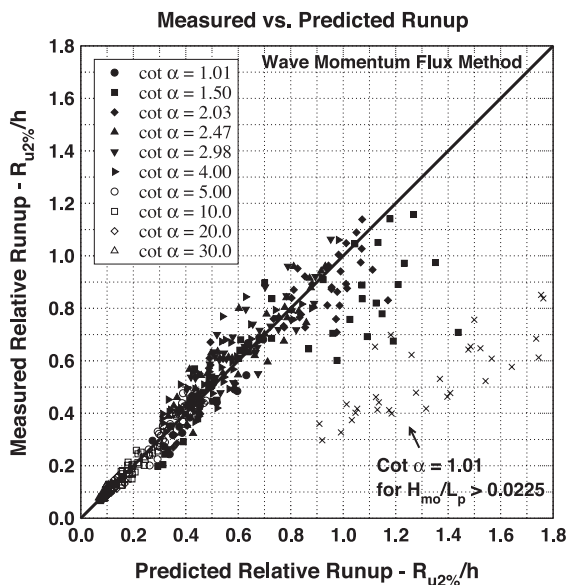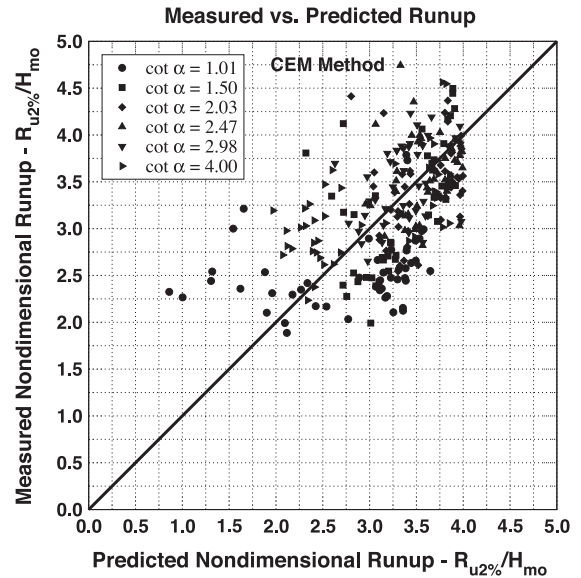

Fig. 10. Comparison of Ahrens' (1981) data to predictions using Eqs. (16) and (17).

quite mild ($\tan\alpha=1/30$) to fairly steep ($\tan\alpha=2/3$). Previously, separate equations based on the Iribarren number were needed to cover this range of slopes for breaking waves. Also note that the slope function, $F(\alpha)$, in Eq. (40) or (41), i.e., $(\tan\alpha)^{0.7}$, has the tangent of the slope angle raised to essentially the same power as given by Mase (1989) in Eq. (20). As seen on Fig. 8, the 0.7 exponent of $\tan\alpha$ lessens the influence of $\tan\alpha$ on run-up as slope decreases. This agrees with the observation of Douglass (1992) that slope angle plays a less important role for wave run-up on mild beaches.

3.4. Solitary wave run-up

Run-up of solitary waves on impermeable plane slopes has been well studied, producing both theoretical/empirical formulas for maximum run-up and numerical models of the entire run-up sequence (e.g., Synolakis, 1986; Li and Raichlen, 2001; Li and Raichlen, 2003). Carrier et al. (2003) crafted an analytical formulation for tsunami run-up, and they noted the location and direction of maximum wave momentum flux for the cases of initial positive and negative wave forms.

Fig. 9. Comparison of Ahrens' (1981) and Mase's (1989) data to predictions using Eqs. (39)–(41).

For a given value of the solitary wave parameter H/h, maximum wave run-up increases for breaking waves as the run-up slope increases. However, when the slope becomes so steep that the waves no longer break, further slope steepening results in decreasing values of run-up. The transition relative wave height between breaking and nonbreaking waves was given by Synolakis (1986) as

$$\left(\frac{H}{h}\right)_{break} = 0.8183(\cot\alpha)^{-10/9} \tag{42}$$

Because of the difference in run-up behavior between breaking and nonbreaking solitary waves, each case is considered separately.

3.4.1. Breaking solitary wave run-up

Measured values of maximum solitary wave run-up for slopes with $\cot\alpha=11.43$, 15.0 and 30.0 were obtained from Hall and Watts (1953), Li (2000) and Briggs et al. (1995), respectively. Corresponding values of the wave momentum flux parameter for solitary waves were calculated for all the data. For each structure slope an empirical coefficient was determined that provided a best fit of Eq. (37). The
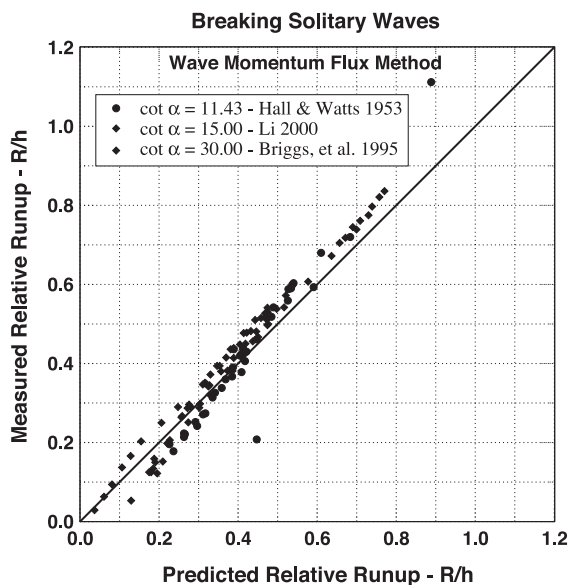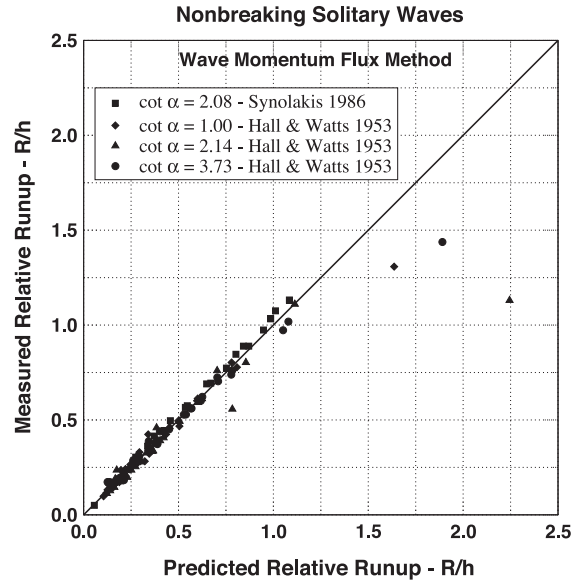

Fig. 12. Measured versus predicted nonbreaking solitary wave run-up—Eq. (44).

coefficients were then expressed as a function of slope resulting in the following simple equation for breaking solitary wave run-up.

$$\frac{R}{h} = (1.39 - 0.027\cot\alpha)\left(\frac{M_F}{\rho g h^2}\right)^{1/2} \tag{43}$$

Fig. 11 plots the 112 measured run-up values versus run-up predicted by Eq. (43). The solid line is the line of equivalence and the overall root-mean-squared error was 0.051. There is reasonable correspondence between estimates and observations, but that was expected because the same data were used to establish the predictive equation. More importantly, there is a bias to the comparison because the simple derivation resulted in the wave momentum flux parameter being raised to the 1/2-power. The data actually showed that a better fit could be obtained if the exponent varied from 1/2 for very mild slopes up to a value of unity near the transition between breaking and nonbreaking waves. This implies that the sea surface of the run-up wedge changes from a nearly straight line to a concave shape as the structure slope increases which agrees with run-up profiles measured by Li and Raichlen (2001). Thus, the simple

Fig. 11. Measured versus predicted breaking solitary wave run-up—Eq. (43).

triangular wedge derivation presented earlier is not appropriate for steeper slopes and a more accurate description of the wedge volume is needed. Nevertheless, Eq. (43) yielded reasonable estimates for slopes between 1/30 and 1/10.

3.4.2. Nonbreaking Solitary Wave Run-up

Nonbreaking wave run-up data from Synolakis (1986, 1987) for a plane impermeable slope with cotα=2.08, and data from Hall and Watts (1953) for slopes with cot α=1.0, 2.14 and 3.73 were used to examine the utility of the wave momentum flux parameter for estimating nonbreaking solitary wave run-up. In this case, relative wave run-up R/h was shown to be directly proportional to the maximum depth-integrated wave momentum flux parameter with a very good fit to the 122 data points provided by the expression

$$\frac{R}{h} = 1.82(\cot\alpha)^{1/5}\left(\frac{M_F}{\rho g h^2}\right) \tag{44}$$

Goodness-of-fit is shown in Fig. 12 for the nonbreaking wave run-up data. With the exception of a few outlying points, the empirical Eq. (44) does remarkably well. Overall root-mean-squared error between measured

and predicted R/h was 0.12 for all data, and the RMS error dropped to 0.034 when the three out-lying points were discarded. Also note that structure slope has a relatively minor influence for nonbreaking solitary waves.

A theoretical run-up equation for nonbreaking solitary waves was presented by Li and Raichlen (2001) as

$$\left(\frac{R}{h}\right)_{Li} = 2.831(\cot\alpha)^{1/2}\left(\frac{H}{h}\right)^{5/4}$$
$$+ 0.293(\cot\alpha)^{3/2}\left(\frac{H}{h}\right)^{9/4} \tag{45}$$

Predictions using Eq. (45) are compared to the nonbreaking run-up data in Fig. 13. Good correspondence is seen except for the milder slope with cotα=3.73. It is interesting to note that substitution of Eq. (31) for $M_F/(\rho g h^2)$ in Eq. (44) results in an expression containing terms with H/h raised to powers that are approximately the same as in Eq. (45).

4. Summary and conclusions

The goal of this study was to develop new formulas for wave run-up on smooth, impermeable plane slopes based on a new parameter representing the maximum depth-integrated wave momentum flux occurring in a wave. These formulas should be as good as existing formulas for estimating run-up due to waves that break on the slope and better at estimating nonbreaking wave run-up.

A crude run-up formula was derived based on the simple argument that the weight of water contained in the run-up wedge above still water level at maximum run-up is proportional to the maximum depth-integrated wave momentum flux in the wave at or near the toe of the slope. The derived general formula included an unknown function of slope that needed to be determined empirically.

Existing published wave run-up data for regular, irregular and solitary waves were used to establish the empirical slope functions for the new wave run-up formulas. Reasonable predictive capability for regular waves was demonstrated for all but the
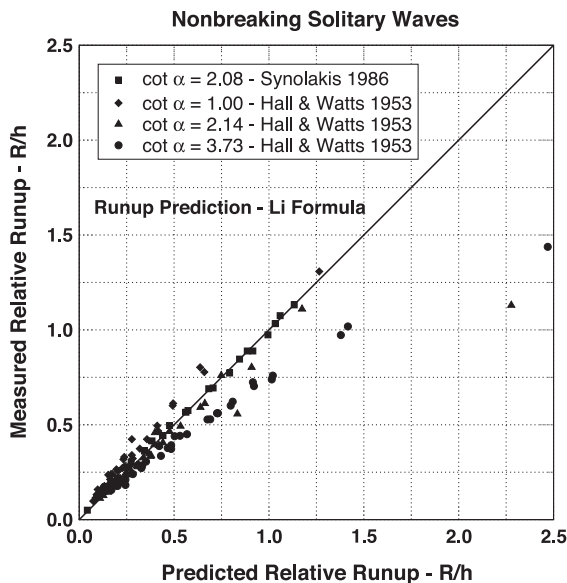

Fig. 13. Measured versus predicted nonbreaking solitary wave run-up—Li formula.

steepest 1:1 slope. Existing regular wave run-up formulas based on Iribarren number did better than the new formula for mild slopes but poorer on steeper slopes.

Irregular wave data from Ahrens (1981) and Mase (1989) were used to establish the empirical slope functions for breaking and nonbreaking wave run-up corresponding to the 2% run-up elevation. The wave momentum flux parameter was defined in terms of the frequency-domain wave parameters H_{mo} and T_p. For waves that break as plunging or spilling breakers on the slope, a single equation was found covering the slopes in the range $1/30 \leq \tan\alpha \leq 2/3$. In this formula, the influence of structure or beach slope on wave run-up decreases with slope in agreement with observations made by Douglass (1992). The formula for nonbreaking surging waves on steep slopes was limited to the range $1/4 \leq \tan\alpha \leq 1/1$. Comparison of predictions to measurements for both breaking and nonbreaking irregular wave run-up were good with the exception of short-period waves breaking on the 1:1 slope. It was hypothesized that the sea surface profile of the run-up wedge was no longer a straight line in this instance, so the crude run-up formula was no longer valid. Estimation of irregular run-up on structure slopes using the formulas given in the Coastal Engineering Manual produced generally poorer comparisons to the measurements of Ahrens (1981).

Maximum run-up of breaking and nonbreaking solitary waves on smooth, impermeable plane slopes was adequately predicted using the wave momentum flux parameter for solitary waves. This illustrates the utility of the wave momentum flux parameter for nonperiodic waves.

The premise that wave run-up can be estimated as a function of the wave momentum flux parameter appears valid based on the data used to develop the empirical formulas in this paper. As noted by Ahrens et al. (1993), there are few irregular wave run-up laboratory data for severe shallow water conditions of near depth-limited breaking ($0.33 \leq H_{mo}/h \leq 0.60$) relative to water depth at the structure toe. So we are not really certain how the laboratory-based wave run-up formulas perform for what may be the design run-up condition. The wave momentum flux parameter includes the effect of

increasing wave nonlinearity, and thus, it is anticipated that the new run-up formulas for irregular waves might give better estimates for very nonlinear waves arriving at the toe of the slope. Likewise, there is hope that the wave momentum flux parameter will prove equally useful for estimating wave run-up on rough and permeable structure slopes. However, both of these hypotheses remain unproven at this time.

Notation

a_1, a_2, a_3 empirical coefficients
A_0 empirical coefficient
A_1 empirical exponent
b_1, b_2 empirical coefficients
c_o empirical coefficient
C empirical coefficient
C_m empirical run-up coefficient
C_p empirical run-up coefficient
C_1, C_2, C_3 empirical coefficients
$F(\alpha)$ empirical function of structure or beach slope
e base of natural logarithm
g gravitational acceleration
h water depth from bottom to the still water level
H uniform steady wave height
H_{limit} steepness limit wave height
H_{mo} zeroth-moment wave height related to the area beneath the spectrum
H_o deepwater uniform wave height
H_s significant wave height for irregular wave train
$H_{1/3}$ average of the highest 1/3 waves in an irregular wave train
k wave number [$=2\pi/L$]
K_M unknown constant of proportionality
K_P reduction factor to account for slope porosity ($K_P=1$ for impermeable slopes)
L local wavelength
L_o deepwater wavelength
L_{om} deepwater wavelength associated with mean irregular wave period T_m
L_{op} deepwater wavelength associated with peak spectral period T_p
L_p wavelength associated with peak spectral period T_p
M coefficient for solitary wave theory (function of H/h)

M_F　depth-integrated wave momentum flux across a unit width

$(M_F)_{max}$　maximum depth-integrated wave momentum flux across a unit width

N　coefficient for solitary wave theory (function of H/h)

P　exceedence probability

P_d　instantaneous wave dynamic pressure at a specified position

r　dimensionless water depth [$=\omega^2 h/g$]

R　maximum vertical run-up from SWL

$\bar{R}$　mean wave run-up elevation

R_{max}　maximum wave run-up elevation

R_P　wave run-up elevation associated with exceedence probability, P

R_s　significant wave run-up elevation

R_X　wave run-up elevation associated with different elevations

$R_{u2\%}$　wave run-up elevation exceeded by highest 2% of run-ups

$R_{1/3}$　average of the highest 1/3 wave run-up elevations

$R_{1/10}$　average of the highest 1/10 wave run-up elevations

S_{xx}　wave-averaged momentum flux (also known as radiation stress)

t　time

T　wave period

T_p　wave period associated with the spectrum peak frequency

T_m　mean wave period in irregular wave train

u　instantaneous horizontal water velocity at a specified position

$W_{(ABC)}$　weight of water per unit crest width in area ABC

x　horizontal coordinate positive in the direction of wave propagation

X_p　depth function in run-up formula

z　vertical coordinate directed positive upward with origin at the SWL

Greek symbols

α　beach or structure slope

η　instantaneous sea surface elevation relative to still water level

θ　unknown angle between still water level and run-up water surface

ξ_o　deepwater Iribarren number

ξ_{om}　deepwater Iribarren number based on T_m and local $H_{1/3}$

ξ_{op}　deepwater Iribarren number based on T_p and local H_{mo}

ξ'_{op}　deepwater Iribarren number based on T_p and local $H_{1/3}$

π　mathematical P_I

Π　wave function in run-up formula

ρ　mass density of water

ω　circular wave frequency [$=2\pi/T$]

Acknowledgements

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